Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

1. A railway truck S of mass 20 tonnes is moving along a straight horizontal track when it collides with another railway truck T of mass 30 tonnes which is at rest. Immediately before the collision the speed of S is $4\,\text{m s}^{-1}$.
As a result of the collision, the two railway trucks join together.

Find

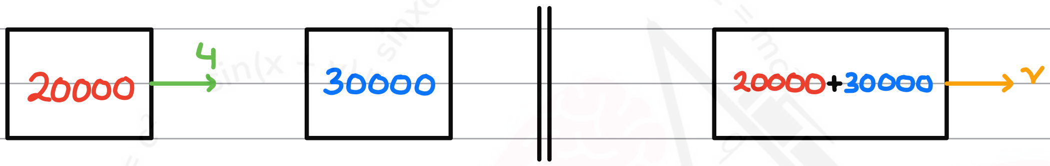
(a) the common speed of the railway trucks immediately after the collision,

(2)

(b) the magnitude of the impulse exerted on S in the collision, stating the units of your answer.

(3)

a) Draw a diagram labelling speeds and masses.



We can now use the law of conservation of momentum where :

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

$\therefore\ 20000 \times 4 + 30000 \times 0 = (20000 + 30000) \times v$

$\therefore\ 80000 = 50000v$

$\therefore\ v = \dfrac{80000}{50000} = \dfrac{8}{5} = 1.6\,\text{ms}^{-1}$

b) The formula for impulse on an object is : $\boxed{I = mv - mu}$

$\therefore\ I = 20000(v) - 20000(4)$

$= 20000(1.6) - 20000(4)$

$= 32000 - 80000$

$= -48000$

$\therefore\ |I| = 48000\,\text{Ns}$

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Question 1 continued

Q1

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2.

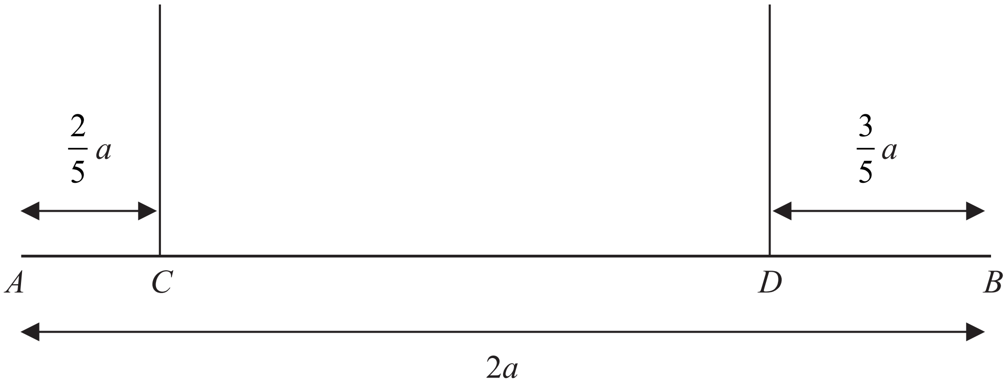


Figure 1

A uniform rod AB has length $2a$ and mass M. The rod is held in equilibrium in a horizontal position by two vertical light strings which are attached to the rod at C and D, where $AC = \frac{2}{5}a$ and $DB = \frac{3}{5}a$, as shown in Figure 1.

A particle P is placed on the rod at B.

The rod remains horizontal and in equilibrium.

(a) Find, in terms of M, the largest possible mass of the particle P

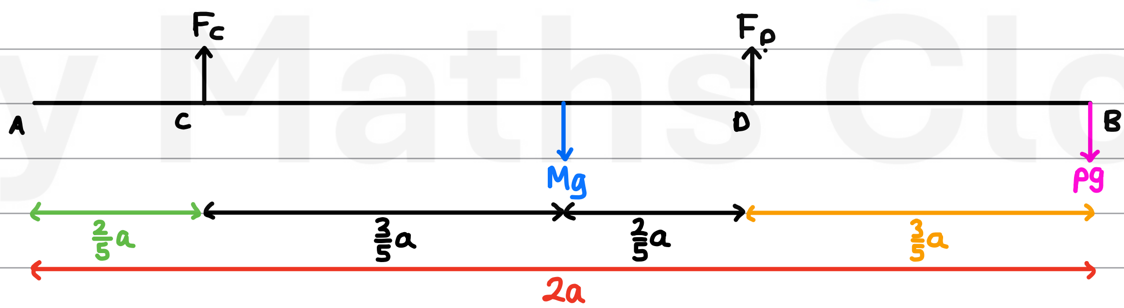
(3)

Given that the mass of P is $\frac{1}{2}M$

(b) find, in terms of M and g, the tension in the string that is attached to the rod at C.

(3)

a) Draw a diagram labelling forces and lengths.



Since it's stated in the question that the beam is in equilibrium, the sum of the clockwise moments is equal to the sum of the anticlockwise moments, therefore :

$\boxed{\Sigma \text{ moments clockwise} = \Sigma \text{ moments anticlockwise}}$ where $\boxed{\text{moment} = \text{force} \times \text{perpendicular distance}}$

Question 2 continued

The clockwise forces are ones that go upwards from the left or downwards from the right of where moments are taken and anticlockwise forces are ones that go upwards from the right or downwards from the left of where moments are taken.

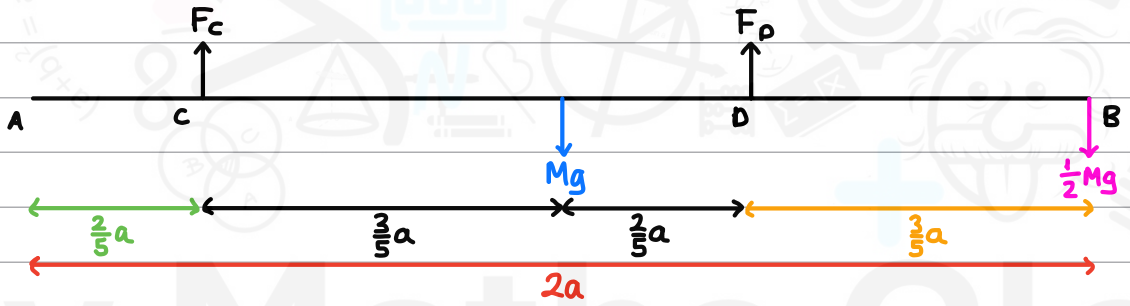
When the mass of P is at its maximum, the rod will be on the point of tipping about D meaning $F_c = 0$. Taking moments around D :

$$\underbrace{pg\left(\tfrac{3}{5}a\right)}_{\text{clockwise moments}} = \underbrace{Mg\left(\tfrac{2}{5}a\right)}_{\text{anticlockwise moments}}$$

$\therefore \frac{3a}{5}pg = \frac{2a}{5}Mg$

$\therefore 3ap = 2aM \quad \therefore p = \frac{2aM}{3a} = \frac{2}{3}M$ //

b) Redraw diagram with new force.



Taking moments around D to eliminate F_D :

$$\underbrace{\tfrac{1}{2}Mg\left(\tfrac{3}{5}a\right) + F_c\left(\tfrac{2}{5}a+\tfrac{3}{5}a\right)}_{\text{clockwise moments}} = \underbrace{Mg\left(\tfrac{2}{5}a\right)}_{\text{anticlockwise moments}}$$

$\therefore \frac{3}{10}Mga + F_c a = \frac{2}{5}Mga$

$F_c a = \frac{2}{5}Mga - \frac{3}{10}Mga = \frac{1}{10}Mga \quad \therefore F_c = \frac{1}{10}Mga \div a = \frac{1}{10}Mg$

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Question 2 continued

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Question 2 continued

Q2

(Total 6 marks)

3.

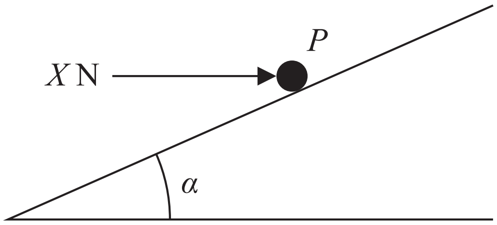


Figure 2

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$

A particle P of mass 2 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 2. The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

(a) Show that when $X = 14.7$ there is no frictional force acting on P

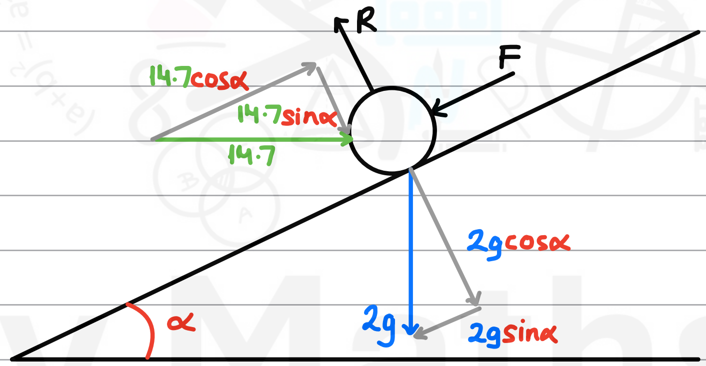
(3)

The coefficient of friction between P and the plane is 0.5

(b) Find the smallest possible value of X.

(8)

a) Draw a diagram labelling all the forces.



$\tan\alpha = \frac{3}{4}$

$\therefore \sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

Since the particle is held in equilibrium, the sum of the forces parallel and perpendicular to the plane is zero.

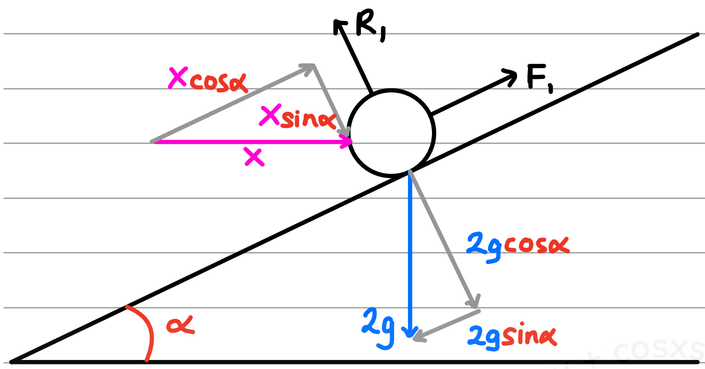
Parallel : $14.7\cos\alpha - 2g\sin\alpha - F = 0 \quad \therefore F = 14.7\cos\alpha - 2g\sin\alpha$

$\therefore F = 14.7\left(\frac{4}{5}\right) - 2(9.8)\left(\frac{3}{5}\right) = 0 \quad \therefore F = 0$ when $X = 14.7$

P 7 2 1 5 1 A 0 8 3 2

Question 3 continued

b) Redraw the diagram when $F > 0$.



Since X will be a constant force, the particle will be in equilibrium.

Using the formula : $\boxed{F = \mu R}$

$\therefore F_1 = 0.5R_1$

Since the particle is held in equilibrium, the sum of the forces parallel and perpendicular to the plane is zero.

Parallel : $X\cos\alpha + F_1 - 2g\sin\alpha = 0 \quad \therefore X\cos\alpha = 2g\sin\alpha - F_1 = 2g\sin\alpha - 0.5R_1$

Perpendicular : $R_1 - X\sin\alpha - 2g\sin\alpha = 0 \quad \therefore R_1 = 2g\cos\alpha + X\sin\alpha$

$\therefore X\cos\alpha = 2g\sin\alpha - 0.5(2g\cos\alpha + X\sin\alpha)$

$X\cos\alpha = 2g\sin\alpha - g\cos\alpha - 0.5X\sin\alpha$

$X\cos\alpha + 0.5X\sin\alpha = 2g\sin\alpha - g\cos\alpha$

$X(\cos\alpha + 0.5\sin\alpha) = 2g\sin\alpha - g\cos\alpha$

$$X = \frac{2g\sin\alpha - g\cos\alpha}{\cos\alpha + 0.5\sin\alpha} = \frac{2(9.8)\left(\frac{3}{5}\right) - 9.8\left(\frac{4}{5}\right)}{\left(\frac{4}{5}\right) + 0.5\left(\frac{3}{5}\right)} = \frac{196}{55} \approx 3.56\text{ N} \quad (3sf)$$

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Question 3 continued

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Question 3 continued

Q3

(Total 11 marks)

4.

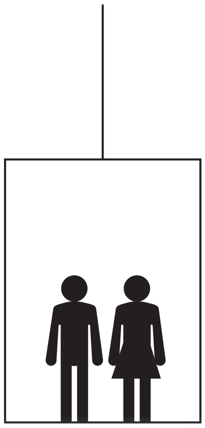


Figure 3

Two children, Alan and Bhavana, are standing on the horizontal floor of a lift, as shown in Figure 3.

The lift has mass 250 kg. The lift is raised vertically upwards with constant acceleration by a vertical cable which is attached to the top of the lift. The cable is modelled as being light and inextensible. While the lift is accelerating upwards, the tension in the cable is 3616 N.

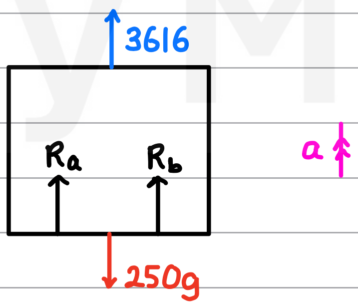
As the lift accelerates upwards, the floor of the lift exerts a force of magnitude 565 N on Alan and a force of magnitude 226 N on Bhavana.

Air resistance is modelled as being negligible and Alan and Bhavana are modelled as particles.

(a) By considering the forces acting on the lift only, find the acceleration of the lift. **(3)**

(b) Find the mass of Alan. **(3)**

a) Draw a diagram labelling forces on the lift only.



Because the lift exerts forces on Alan and Bhavana, they also exert an equal and opposite reaction force on the lift.

$\therefore R_a = 565$, $R_b = 226$

Using the formula : $\boxed{\Sigma F = ma}$ taking upwards as positive ,

$3616 - 250g - 565 - 226 = 250a$

$\therefore 250a = 375$ $\therefore a = \frac{375}{250} = 1.5\ ms^{-2}$

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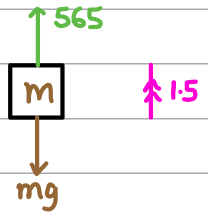
Question 4 continued

b) Consider forces only on Alan.



Using $\boxed{\Sigma F = ma}$ taking upwards as positive:

$565 - mg = m(1.5)$ $\therefore\ 1.5m + (9.8)m = 565$

$\therefore\ m(1.5+9.8) = 565$ $\therefore\ m = \dfrac{565}{1.5+9.8} = 50\text{ kg}$

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Question 4 continued

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Question 4 continued

Q4

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5. A small ball is projected vertically upwards with speed $29.4\,\mathrm{m\,s^{-1}}$ from a point A which is $19.6\,\mathrm{m}$ above horizontal ground.

The ball is modelled as a particle moving freely under gravity until it hits the ground. It is assumed that the ball does not rebound.

(a) Find the distance travelled by the ball while its speed is less than $14.7\,\mathrm{m\,s^{-1}}$

(3)

(b) Find the time for which the ball is moving with a speed of more than $29.4\,\mathrm{m\,s^{-1}}$

(3)

(c) Sketch a speed-time graph for the motion of the ball from the instant when it is projected from A to the instant when it hits the ground. Show clearly where your graph meets the axes.

(3)

a) Since the ball is falling freely under gravity, there is a constant acceleration so SUVAT can be used.

S : s
u : 14.7
v : 0
a : −9.8
t :

Equation without time : $\boxed{v^2 = u^2 + 2as}$

$\therefore 0^2 = 14.7^2 + 2(-9.8)s$

$\therefore 19.6s = 14.7^2 \quad \therefore s = \frac{14.7^2}{19.6} = 11.025$

Since the SUVAT was set up with the end as the point where the ball stops before coming back down, we have only found half the distance so we need to double it.

$\therefore d = 2 \times 11.025 = 22.05 \approx 22.1\,\mathrm{m}$ (3sf)

b) Setting up a SUVAT for the new speed.

S : 19.6
u : 29.4
v :
a : 9.8
t : t

Equation without v : $\boxed{s = ut + \frac{1}{2}at^2}$

$\therefore 19.6 = 29.4t + \frac{1}{2}(9.8)t^2$

$\therefore 4.9t^2 + 29.4t + 19.6 = 0$

$t = -3 \pm \sqrt{13}$ s

$t = 0.606$, $t = -6.61$ $\quad t > 0 \quad \therefore t = 0.606$ (3sf)

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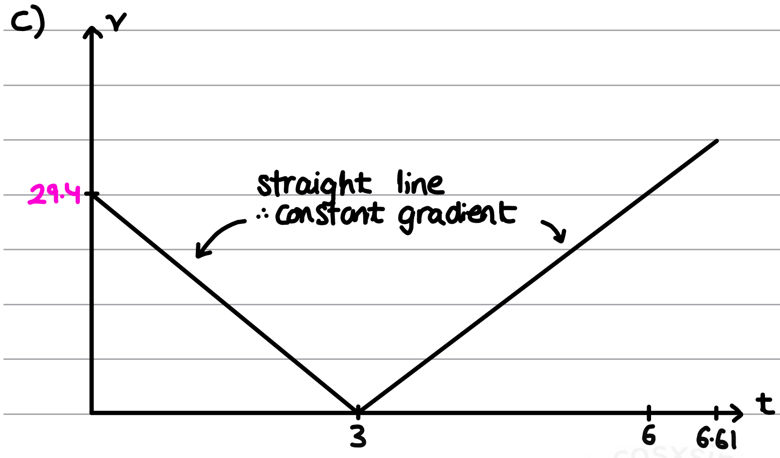
Question 5 continued

c)



Using $\boxed{v = u + at}$:

when $v = 0$, we get that

$t = -\frac{u}{a} = -\frac{29.4}{-9.8} = 3\,s$

for the time when the ball reverses direction.

Set up a SUVAT to find the total time but first calculate the maximum height.

s : s

u : 29.4

v :

a : −9.8

t : 3

Equation without v : $\boxed{s = ut + \frac{1}{2}at^2}$

$\therefore s = 29.4(3) + \frac{1}{2}(-9.8)(3^2) = 44.1$

∴ Total height above ground = $44.1 + 19.6 = 63.7$ m

Now we can find the time to hit the ground.

s : 63.7

u : 0

v :

a : 9.8

t : t

Equation without v : $\boxed{s = ut + \frac{1}{2}at^2}$

$\therefore 63.7 = (0)t + \frac{1}{2}(9.8)t^2$

$\therefore t^2 = \frac{2 \times 63.7}{9.8} = 13 \quad \therefore t = \sqrt{13} = 3.61$ (3sf)

Since the time from $v = 0$ to the end is greater than the time from the start to $v = 0$, line after $v = 0$ will continue for longer.

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Question 5 continued

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Question 5 continued

Q5

(Total 9 marks)

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6. [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle A of mass 0.5 kg is at rest on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (-3\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants, are applied to A.

Given that A moves in the direction of the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) show that $2p + q - 4 = 0$

(4)

Given that $p = 5$

(b) find the speed of A at time $t = 4$ seconds.

(5)

a) It's given that $F_1 + F_2 = i - 2j$

$\therefore (-3+2j) + (pi + qi) = (-3+p)i + (2+q)j = (i-2j)$

Rewrite as a ratio : $\dfrac{-3+p}{2+q} = \dfrac{1}{-2}$ $\therefore$ $6-2p = 2+q$ $\therefore$ $2p+q-4=0$

b) When $p=5$, $q = 4-2(5) = -6$

$\therefore$ Resultant force $= (-3+5)i + (2+-6)j = 2i-4j$

Using $\boxed{\Sigma F = ma}$: $2i-4j = 0.5a$ $\therefore$ $a = 4i-8j$

Using $\boxed{a = \dfrac{\Delta v}{\Delta t}}$: $4i-8j = \dfrac{v}{4}$ $\therefore$ $v = 16i - 32j$ @ $t=4$

$\therefore$ Speed $= |v| = \sqrt{(16)^2 + (-32)^2} = 16\sqrt{5}$ ms^{-1}

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Question 6 continued

Question 6 continued

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Question 6 continued

Q6

(Total 9 marks)

7.

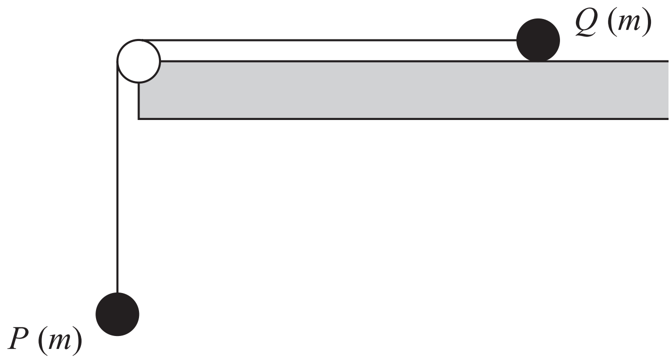


Figure 4

A particle P of mass m is attached to one end of a light inextensible string. Another particle Q, also of mass m, is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at the edge of a rough horizontal table. Particle Q is held at rest on the table and particle P hangs vertically below the pulley with the string taut, as shown in Figure 4.

The pulley, P and Q all lie in the same vertical plane.

The coefficient of friction between Q and the table is μ, where $\mu < 1$

Particle Q is released from rest.

The tension in the string before Q hits the pulley is kmg, where k is a constant.

(a) Find k in terms of μ.

(7)

Given that Q is initially a distance d from the pulley,

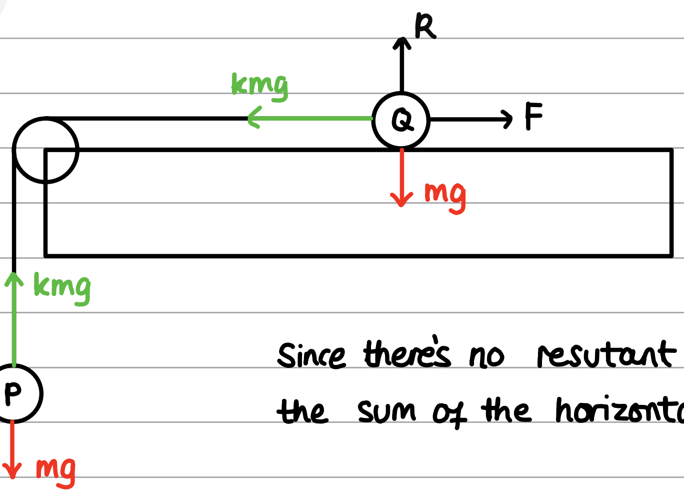
(b) find, in terms of d, g and μ, the time taken by Q, after release, to reach the pulley.

(4)

(c) Describe what would happen if $\mu \geqslant 1$, giving a reason for your answer.

(2)

a) Redraw the diagram with forces labelled.



Using $\boxed{F = \mu R}$ to work out friction

$\therefore F = \mu R = \mu mg$

Since there's no resultant force vertically on Q, the sum of the horizontal forces is zero.

$\therefore R - mg = 0 \quad \therefore R = mg$

Question 7 continued

Equation of forces on P using $\boxed{\Sigma F = ma}$: $mg - kmg = ma$

Equation of forces on Q using $\boxed{\Sigma F = ma}$: $kmg - \mu mg = ma$

Note that whichever way an object moves / accelerates, that is the direction we take as positive.
We can equate the sum of forces as they both equal to 'ma'.

$\therefore mg - kmg = kmg - \mu mg \rightarrow 2kmg = mg + \mu mg$

$\therefore 2k = 1 + \mu \quad \therefore k = \frac{1}{2}(1+\mu)$

b) Since the acceleration is constant we can use SUVAT but we must calculate it using our above equations.

$mg - kmg = ma \quad \therefore a = g - kg = g(1-k) = g\left(1 - \frac{1}{2}(1+\mu)\right) = \frac{g}{2}(1-\mu)$

S : d
u : 0
v :
a : $\frac{g}{2}(1-\mu)$
t : t

Equation without v : $\boxed{S = ut + \frac{1}{2}at^2}$

$\therefore d = (0)t + \frac{1}{2}\left(\frac{g}{2}(1-\mu)\right)t^2$

$d = \frac{g}{4}(1-\mu)t^2 \quad \therefore \frac{4d}{g(1-\mu)} = t^2 \quad \therefore t = \sqrt{\frac{4d}{g(1-\mu)}}$

c) If $\mu \geq 0$ then $a \leq 0$ which isn't possible since we defined the acceleration going upwards for P and towards the pulley for Q. If $a \leq 0$, it implies the frictional force is pulling P towards the pulley which is impossible so $\mu < 1$.

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Question 7 continued

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Question 7 continued

Q7

(Total 13 marks)

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8. [*In this question,* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, A and B, are moving with constant velocities.
The velocity of A is $(3\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and the velocity of B is $(p\mathbf{i} + q\mathbf{j})\,\text{km h}^{-1}$

(a) Find the speed of A.

(2)

The ships are modelled as particles.
At 12 noon, A is at the point with position vector $(-9\mathbf{i} + 6\mathbf{j})\,\text{km}$ and B is at the point with position vector $(16\mathbf{i} + 6\mathbf{j})\,\text{km}$.

At time t hours after 12 noon,

$$\overrightarrow{AB} = [(25 - 12t)\mathbf{i} - 9t\mathbf{j}]\,\text{km}$$

(b) Find the value of p and the value of q.

(7)

(c) Find the bearing of A from B when the ships are 15 km apart, giving your answer to the nearest degree.

(7)

a) $\text{Speed}_A = |v| = \sqrt{3^2 + 12^2} = 3\sqrt{17}\ \text{ms}^{-1}$

b) Form equations of the position of the ships using : $\boxed{\underline{r} = \underline{r}_0 + vt}$

Ship A : $\underline{a} = (-9i + 6j) + t(3i + 12j)$
Ship B : $\underline{b} = (16i + 6j) + t(pi + qj)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (16i + 6j) + t(pi + qj) - [(-9i + 6j) + t(3i + 12j)]$
$= [(16 - -9)i + (6 - 6j)] + t[(p - 3)i + (q - 12)j]$
$= (25 + t(p-3))i + t(q-12)j = \overrightarrow{AB}$

$\therefore (25 + t(p-3))i + t(q-12)j = (25 - 12t)i - 9tj$

Comparing $\underline{i}$ and $\underline{j}$ coefficients :
$\therefore 25 + t(p-3) = 25 - 12t$, $t(q-12) = -9t$
$25 + tp - 3t = 25 - 12t$ $\quad tq - 12t = -9t$
$tp = -9t$ $\quad tq = 3t$
$p = -9$ $\quad q = 3$

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Question 8 continued

c) When ships are 15km apart : $|\vec{AB}| = 15$

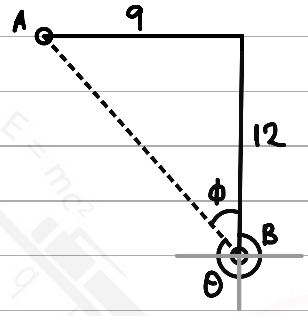
$\therefore \sqrt{(25-12t)^2 + (-9t)^2} = 15 \quad \therefore (25-12t)^2 + (-9t)^2 = 15^2$

$\therefore 625 - 600t + 144t^2 + 81t^2 = 225$

$225t^2 - 600t + 400 = 0$

$t = \frac{4}{3}$

$\therefore \vec{AB} = (25 - 12(\frac{4}{3}))i - 9(\frac{4}{3})j = 9i - 12j \rightarrow$



$\therefore \tan\phi = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{12} \quad \therefore \phi = 36.869 \approx 37^\circ$

$\therefore$ Bearing $(\theta) = 360 - 37 = 323^\circ$

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Question 8 continued

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Question 8 continued

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Q8

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

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